

# Engineering Mechanics: Statics in SI Units, 12e

**3**

**Equilibrium of a Particle**

# Chapter Objectives

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- Concept of the free-body diagram for a particle
- Solve particle equilibrium problems using the equations of equilibrium

# Chapter Outline

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1. Condition for the Equilibrium of a Particle
2. The Free-Body Diagram
3. Coplanar Systems
4. Three-Dimensional Force Systems

# 3.1 Condition for the Equilibrium of a Particle

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- Particle at *equilibrium* if
  - At rest
  - Moving at constant a constant velocity
- Newton's first law of motion

$$\sum \mathbf{F} = 0$$

where  $\sum \mathbf{F}$  is the vector sum of all the forces acting on the particle

## 3.1 Condition for the Equilibrium of a Particle

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- Newton's second law of motion

$$\sum \mathbf{F} = m\mathbf{a}$$

- When the force fulfill Newton's first law of motion,

$$m\mathbf{a} = 0$$

$$\mathbf{a} = 0$$

therefore, the particle is moving in constant velocity or at rest

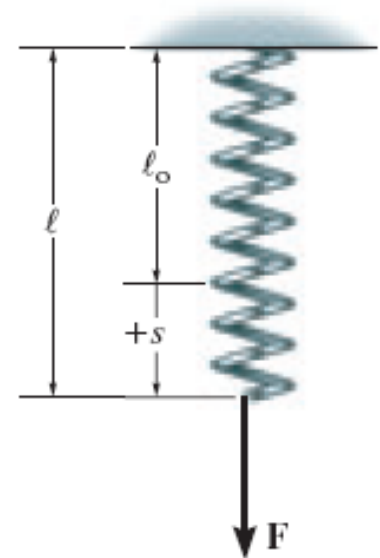
## 3.2 The Free-Body Diagram

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- Best representation of all the unknown forces ( $\sum \mathbf{F}$ ) which acts on a body
- A sketch showing the particle “free” from the surroundings with all the forces acting on it
- Consider two common connections in this subject –
  - Spring
  - Cables and Pulleys

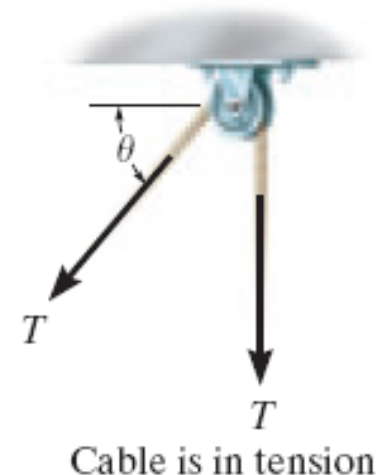
## 3.2 The Free-Body Diagram

- Spring
  - Linear elastic spring: *change in length is directly proportional to the force acting on it*
  - *spring constant or **stiffness**  $k$* : defines the elasticity of the spring
  - Magnitude of force when spring is elongated or compressed  
→  $F = ks$



## 3.2 The Free-Body Diagram

- Cables and Pulley
  - Cables (or cords) are assumed negligible weight and cannot stretch
  - Tension always acts in the direction of the cable
  - Tension force must have a constant magnitude for equilibrium
  - For any angle  $\theta$ , the cable is subjected to a constant tension  $T$





## 3.2 The Free-Body Diagram

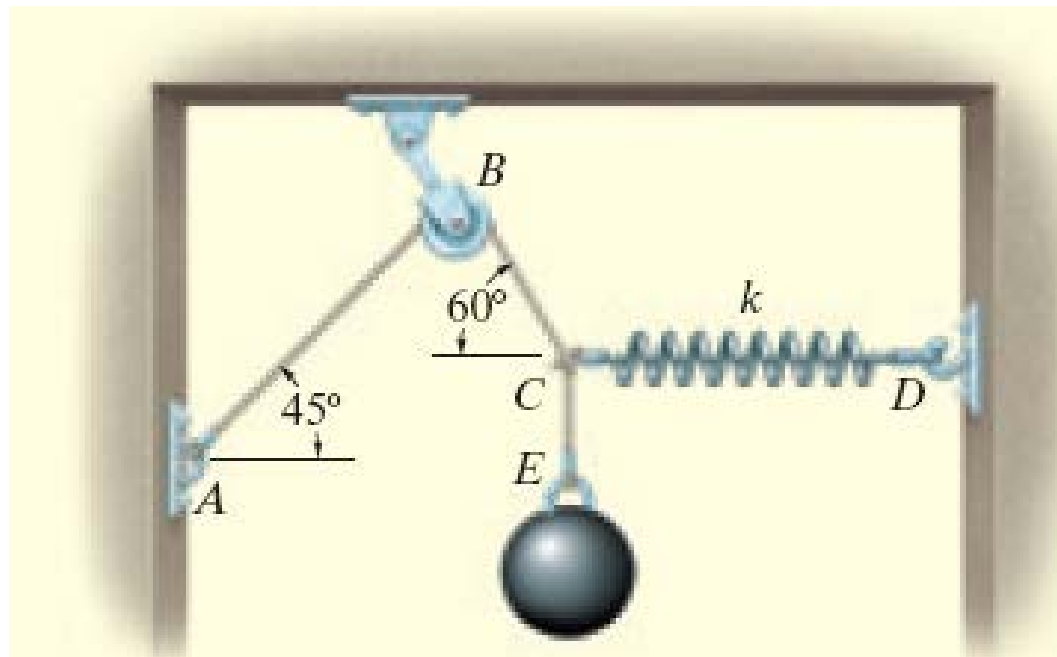
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### Procedure for Drawing a FBD

1. Draw outlined shape
2. Show all the forces
  - Active forces: particle in motion
  - Reactive forces: constraints that prevent motion
3. Identify each forces
  - Known forces with proper magnitude and direction
  - Letters used to represent magnitude and directions of forces that are unknown.

## Example 3.1

The sphere has a mass of 6kg and is supported. Draw a free-body diagram of the sphere, the cord CE and the knot at C.

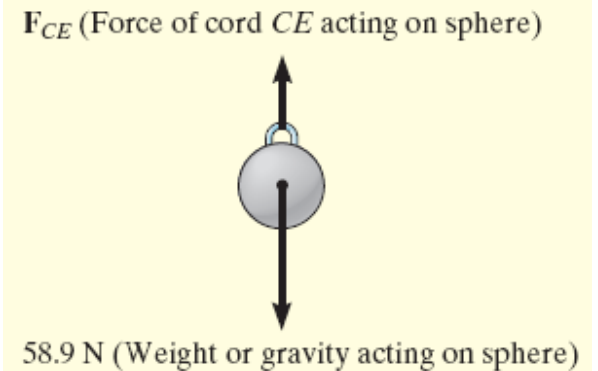


# Solution

## FBD at Sphere

Two forces acting, weight and the force on cord CE.

Weight of 6kg ( $9.81\text{m/s}^2$ ) = 58.9N



## Cord CE

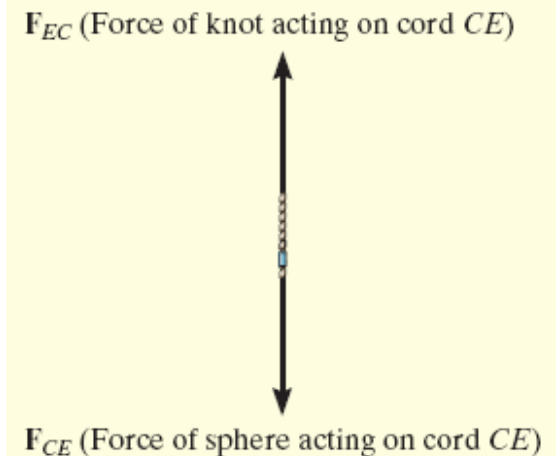
Two forces acting: sphere and knot

Newton's 3<sup>rd</sup> Law:

$F_{CE}$  is equal but opposite

$F_{CE}$  and  $F_{EC}$  pull the cord in tension

For equilibrium,  $F_{CE} = F_{EC}$

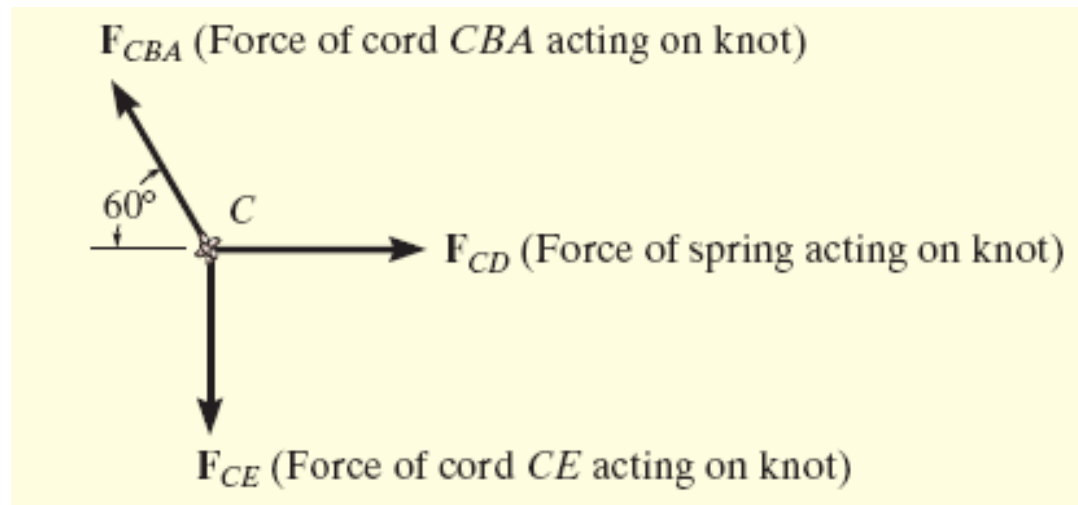


# Solution

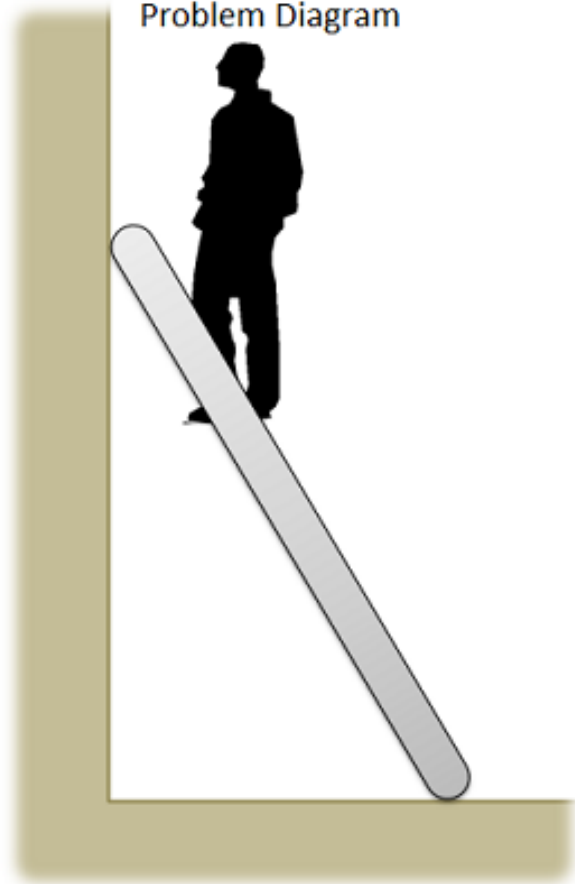
## FBD at Knot

3 forces acting: cord C, i.e., BA, cord CE and spring CD

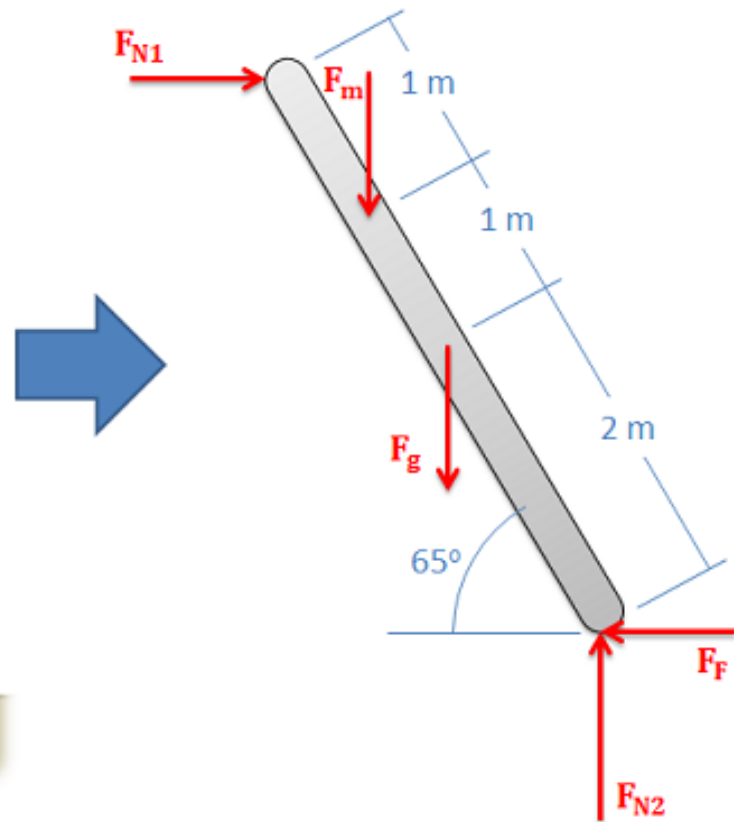
Important to know that the weight of the sphere does not act directly on the knot but subjected to by the cord CE

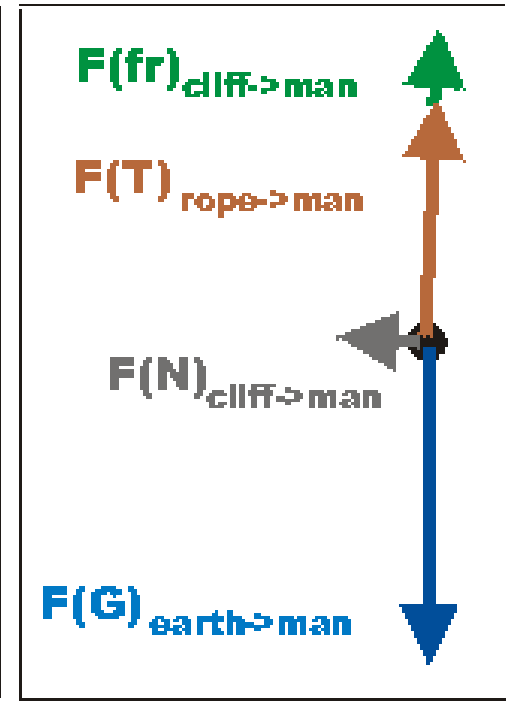
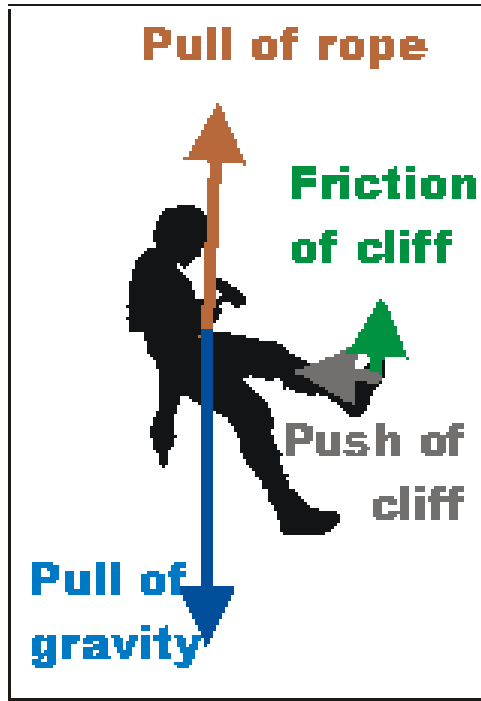


Problem Diagram



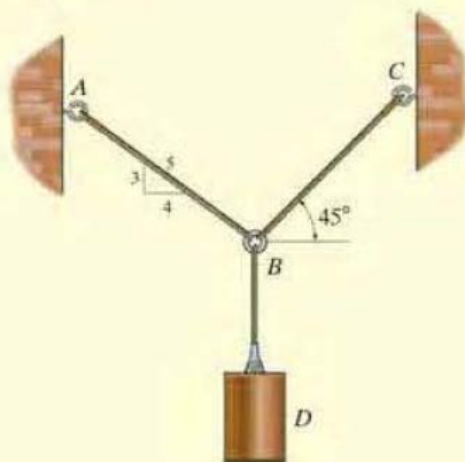
Free Body Diagram





### EXAMPLE 3.2

Determine the tension in cables  $BA$  and  $BC$  necessary to support the 60-kg cylinder in Fig. 3-6a.



(a)

$$T_{BD} = 60(9.81) \text{ N}$$



$$60(9.81) \text{ N}$$

(b)

**Free-Body Diagram.** Due to equilibrium, the weight of the cylinder causes the tension in cable  $BD$  to be  $T_{BD} = 60(9.81) \text{ N}$ , Fig. 3-6b. The forces in cables  $BA$  and  $BC$  can be determined by investigating the equilibrium of ring  $B$ . Its free-body diagram is shown in Fig. 3-6c. The magnitudes of  $\mathbf{T}_A$  and  $\mathbf{T}_C$  are unknown, but their directions are known.

**Equations of Equilibrium.** Applying the equations of equilibrium along the  $x$  and  $y$  axes, we have

$$\rightarrow \Sigma F_x = 0; \quad T_C \cos 45^\circ - \left(\frac{4}{5}\right)T_A = 0 \quad (1)$$

$$+ \uparrow \Sigma F_y = 0; \quad T_C \sin 45^\circ + \left(\frac{3}{5}\right)T_A - 60(9.81) \text{ N} = 0 \quad (2)$$

Equation (1) can be written as  $T_A = 0.8839T_C$ . Substituting this into Eq. (2) yields

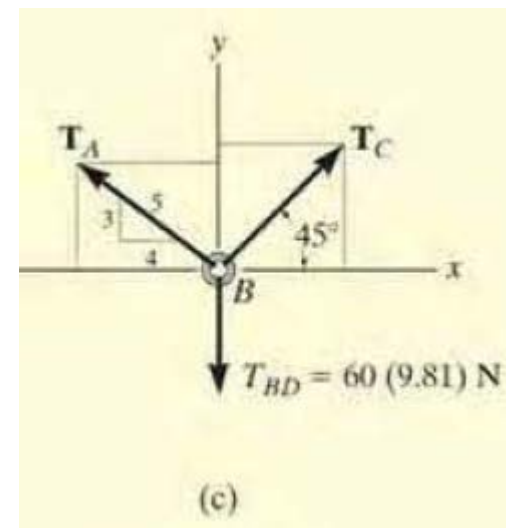
$$T_C \sin 45^\circ + \left(\frac{3}{5}\right)(0.8839T_C) - 60(9.81) \text{ N} = 0$$

So that

$$T_C = 475.66 \text{ N} = 476 \text{ N} \quad \text{Ans.}$$

Substituting this result into either Eq. (1) or Eq. (2), we get

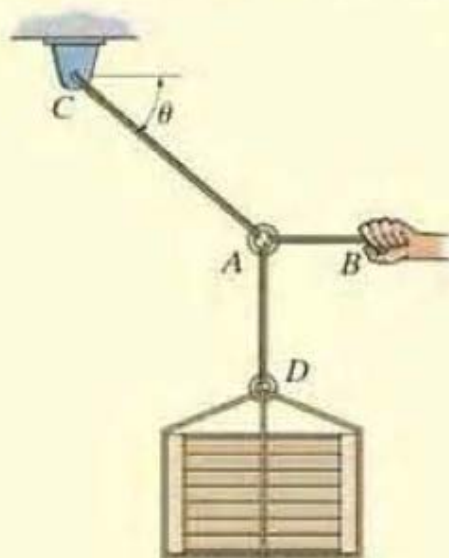
$$T_A = 420 \text{ N} \quad \text{Ans.}$$



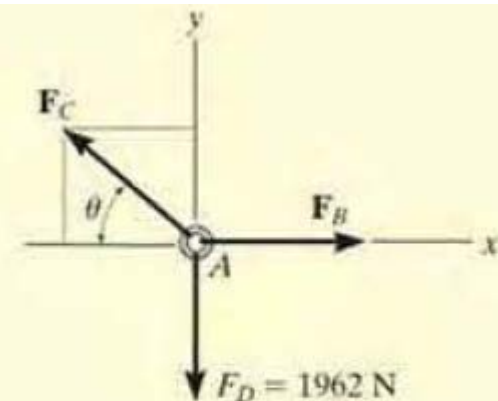


### EXAMPLE 3.3

The 200-kg crate in Fig. 3-7a is suspended using the ropes  $AB$  and  $AC$ . Each rope can withstand a maximum force of 10 kN before it breaks. If  $AB$  always remains horizontal, determine the smallest angle  $\theta$  to which the crate can be suspended before one of the ropes breaks.



(a)



(b)

**Free-Body Diagram.** We will study the equilibrium of ring  $A$ . There are three forces acting on it, Fig. 3-7b. The magnitude of  $F_D$  is equal to the weight of the crate, i.e.,  $F_D = 200(9.81) \text{ N} = 1962 \text{ N} < 10 \text{ kN}$ .

**Equations of Equilibrium.** Applying the equations of equilibrium along the  $x$  and  $y$  axes,

$$\rightarrow \Sigma F_x = 0; \quad -F_C \cos \theta + F_B = 0; \quad F_C = \frac{F_B}{\cos \theta} \quad (1)$$

$$+ \uparrow \Sigma F_y = 0; \quad F_C \sin \theta - 1962 \text{ N} = 0 \quad (2)$$

From Eq. (1),  $F_C$  is always greater than  $F_B$  since  $\cos \theta \leq 1$ . Therefore, rope  $AC$  will reach the maximum tensile force of 10 kN *before* rope  $AB$ . Substituting  $F_C = 10 \text{ kN}$  into Eq. (2), we get

$$[10(10^3) \text{ N}] \sin \theta - 1962 \text{ N} = 0$$

$$\theta = \sin^{-1}(0.1962) = 11.31^\circ = 11.3^\circ \quad \text{Ans.}$$

The force developed in rope  $AB$  can be obtained by substituting the values for  $\theta$  and  $F_C$  into Eq. (1).

$$10(10^3) \text{ N} = \frac{F_B}{\cos 11.31^\circ}$$
$$F_B = 9.81 \text{ kN}$$

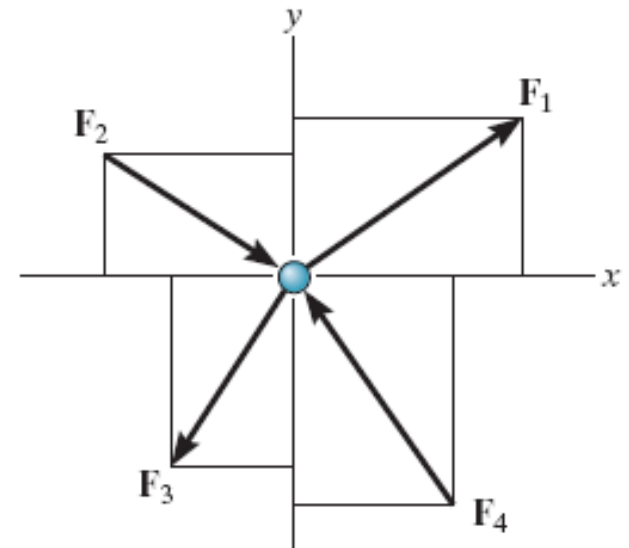
## 3.3 Coplanar Systems

- A particle is subjected to coplanar forces in the x-y plane
- Resolve into **i** and **j** components for equilibrium

$$\sum \mathbf{F}_x = 0$$

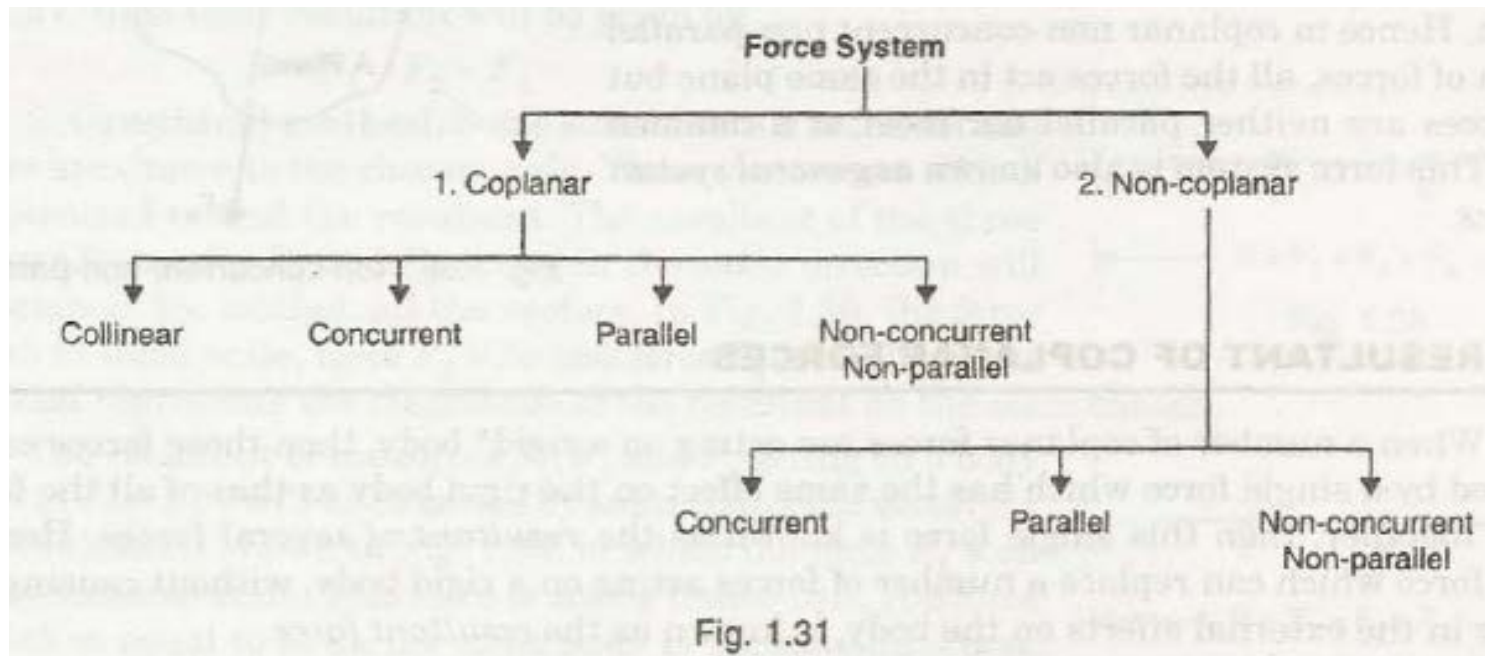
$$\sum \mathbf{F}_y = 0$$

- Scalar equations of equilibrium require that the algebraic sum of the x and y components to equal to zero

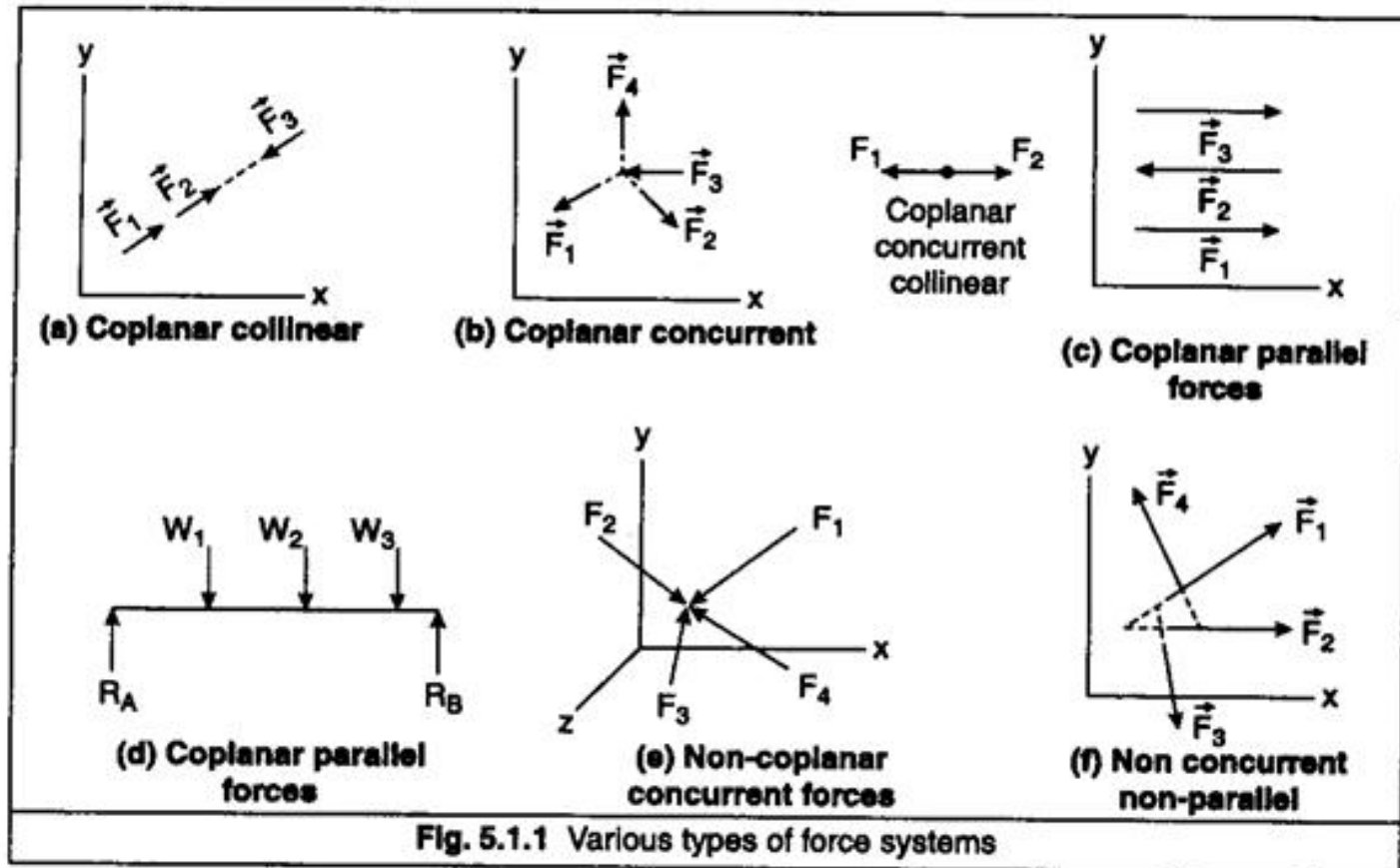


## 3.3 Coplanar Systems

**Coplanar forces** means the **forces** in a plane. When several **forces** act on a body, then they are called a **force system** or a **system of forces**. In a **system** in which all the **forces** lie in the same plane, it is known as **coplanar force system**.



## 3.3 Coplanar Systems



## 3.3 Coplanar Systems

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- Procedure for Analysis

1. Free-Body Diagram

- Establish the x, y axes
- Label all the unknown and known forces

2. Equations of Equilibrium

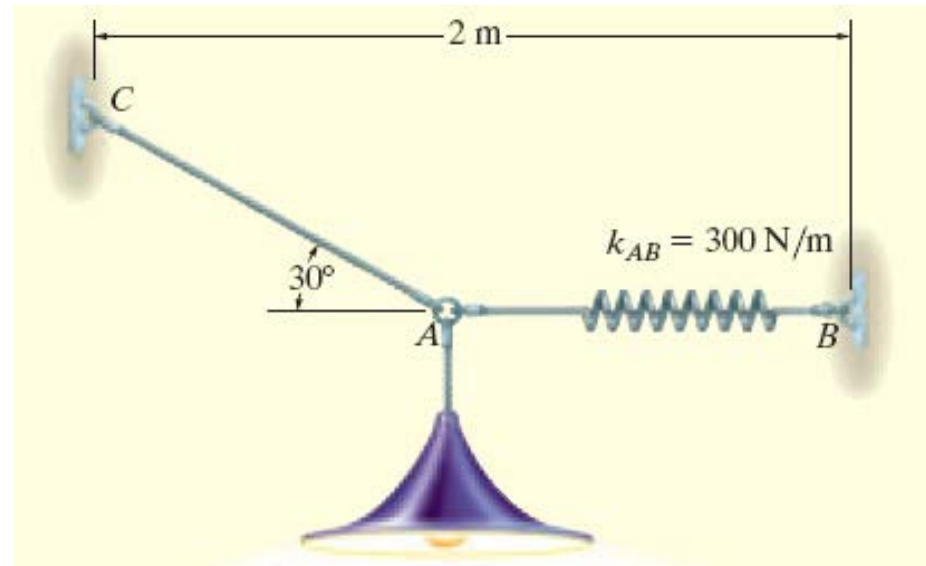
- Apply  $F = ks$  to find spring force
- When negative result force is the reverse
- Apply the equations of equilibrium

$$\sum F_x = 0$$

$$\sum F_y = 0$$

## Example 3.4

Determine the required length of the cord AC so that the 8kg lamp is suspended. The undeformed length of the spring AB is  $l'_{AB} = 0.4\text{m}$ , and the spring has a stiffness of  $k_{AB} = 300\text{N/m}$ .



# Solution

## FBD at Point A

Three forces acting, force by cable AC, force in spring AB and weight of the lamp.

If force on cable AB is known, stretch of the spring is found by  $F = ks$ .

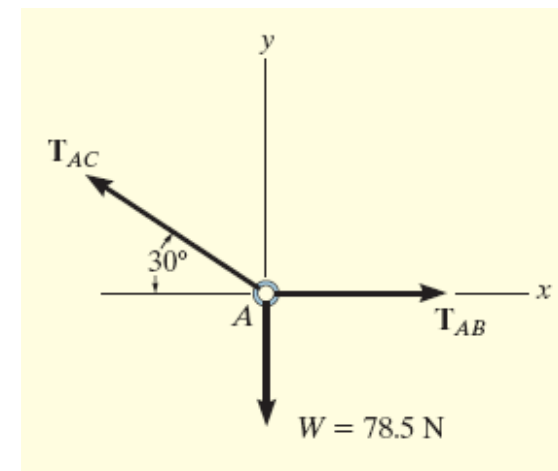
$$+\rightarrow \quad \sum \mathbf{F}_x = 0; \quad T_{AB} - T_{AC} \cos 30^\circ = 0$$

$$+\uparrow \quad \sum \mathbf{F}_y = 0; \quad T_{AC} \sin 30^\circ - 78.5 \text{ N} = 0$$

Solving,

$$T_{AC} = 157.0 \text{ kN}$$

$$T_{AB} = 136.0 \text{ kN}$$





# Solution

$$T_{AB} = k_{AB}s_{AB}; 136.0N = 300N/m(s_{AB})$$

$$s_{AB} = 0.453m$$

For stretched length,

$$l_{AB} = l'_{AB} + s_{AB}$$

$$\begin{aligned} l_{AB} &= 0.4m + 0.453m \\ &= 0.853m \end{aligned}$$

For horizontal distance BC,

$$2m = l_{AC}\cos 30^{\circ} + 0.853m$$

$$l_{AC} = 1.32m$$

## 3.4 Three-Dimensional Force Systems

- For particle equilibrium

$$\sum \mathbf{F} = 0$$

- Resolving into  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$  components

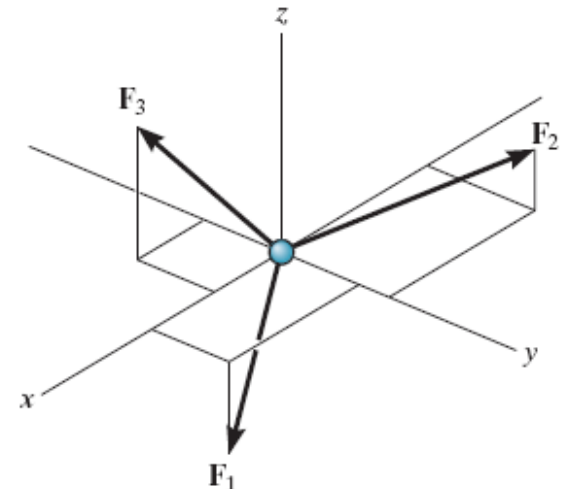
$$\sum F_x \mathbf{i} + \sum F_y \mathbf{j} + \sum F_z \mathbf{k} = 0$$

- Three scalar equations representing algebraic sums of the  $x$ ,  $y$ ,  $z$  forces

$$\sum F_x \mathbf{i} = 0$$

$$\sum F_y \mathbf{j} = 0$$

$$\sum F_z \mathbf{k} = 0$$



## 3.4 Three-Dimensional Force Systems

- Procedure for Analysis

### Free-body Diagram

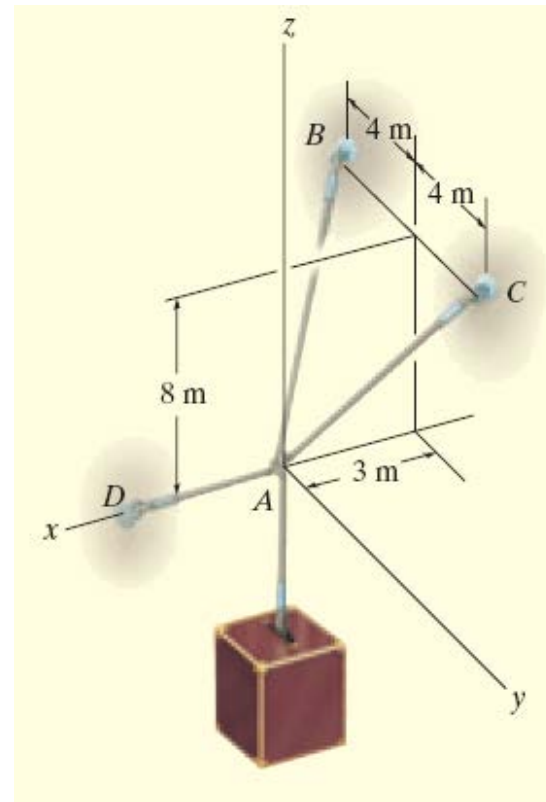
- Establish the x, y, z axes
- Label all known and unknown force

### Equations of Equilibrium

- Apply  $\sum F_x = 0$ ,  $\sum F_y = 0$  and  $\sum F_z = 0$
- Substitute vectors into  $\sum \mathbf{F} = 0$  and set i, j, k components = 0
- Negative results indicate that the sense of the force is opposite to that shown in the FBD.

## Example 3.7

Determine the force developed in each cable used to support the 40kN crate.



# Solution

## FBD at Point A

To expose all three unknown forces in the cables.

## Equations of Equilibrium

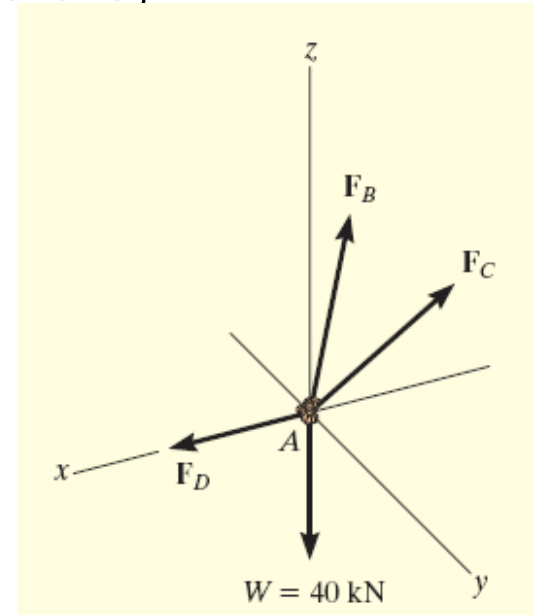
Expressing each forces in Cartesian vectors,

$$\begin{aligned}\mathbf{F}_B &= F_B(\mathbf{r}_B / r_B) \\ &= -0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{F}_C &= F_C(\mathbf{r}_C / r_C) \\ &= -0.318F_C\mathbf{i} + 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k}\end{aligned}$$

$$\mathbf{F}_D = F_D\mathbf{i}$$

$$\mathbf{W} = -40\mathbf{k}$$



# Solution

For equilibrium,

$$\begin{aligned}\sum \mathbf{F} = 0; \quad & \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = 0 \\ & -0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k} - 0.318F_C\mathbf{i} \\ & + 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k} + F_D\mathbf{i} - 40\mathbf{k} = 0\end{aligned}$$

$$\sum F_x = 0; \quad -0.318F_B - 0.318F_C + F_D = 0$$

$$\sum F_y = 0; \quad -0.424F_B + 0.424F_C = 0$$

$$\sum F_z = 0; \quad 0.848F_B + 0.848F_C - 40 = 0$$

Solving,

$$F_B = F_C = 23.6\text{kN}$$

$$F_D = 15.0\text{kN}$$